Gradient Domain Imaging

Introduction & Poisson equation

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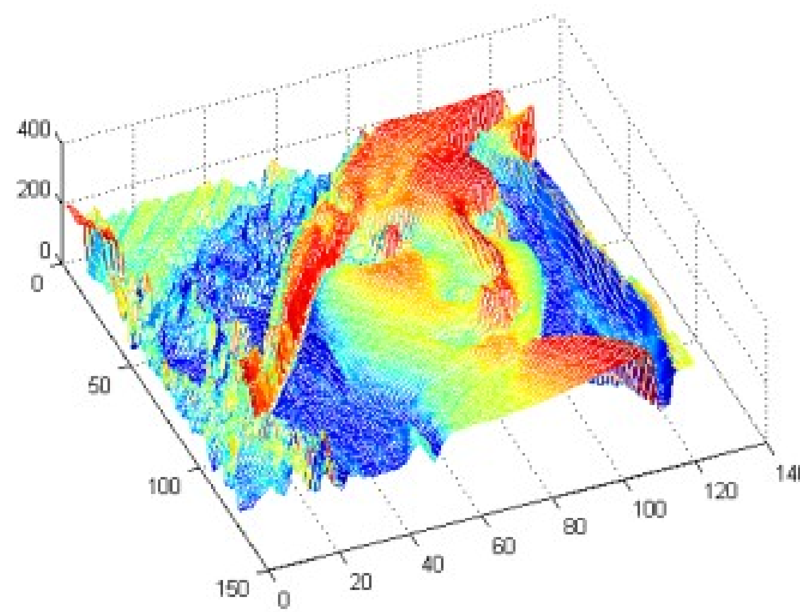
Scalar- vs. Gradient-field

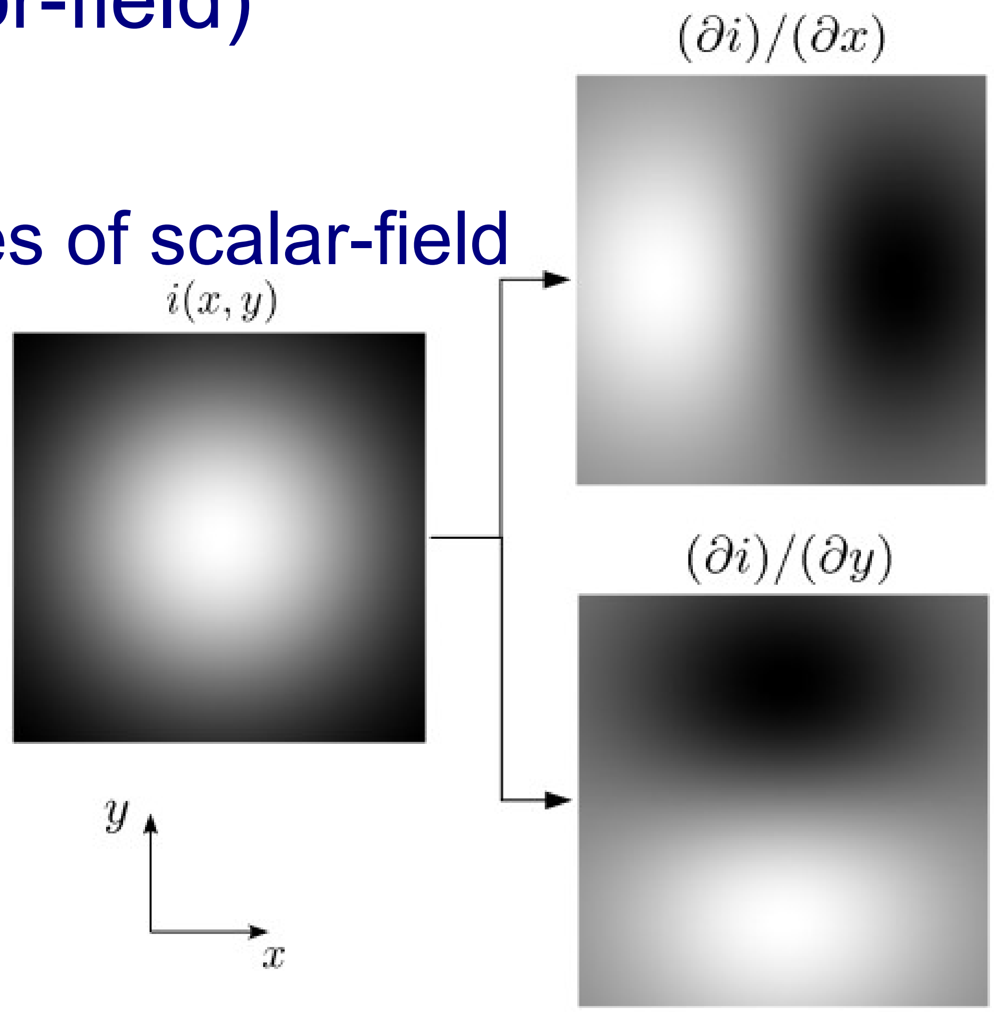
- Scalar-field : classic ordinary images
  - RGB / Gray-scale
  - Every point (pixel) in the image space has a scalar (value)



Scalar- vs. Gradient-field (2)

- Gradient-field (vector-field)
  - 2 components
  - Partial derivatives of scalar-field

$I(x, y)$ • $\nabla I = \{\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\}$

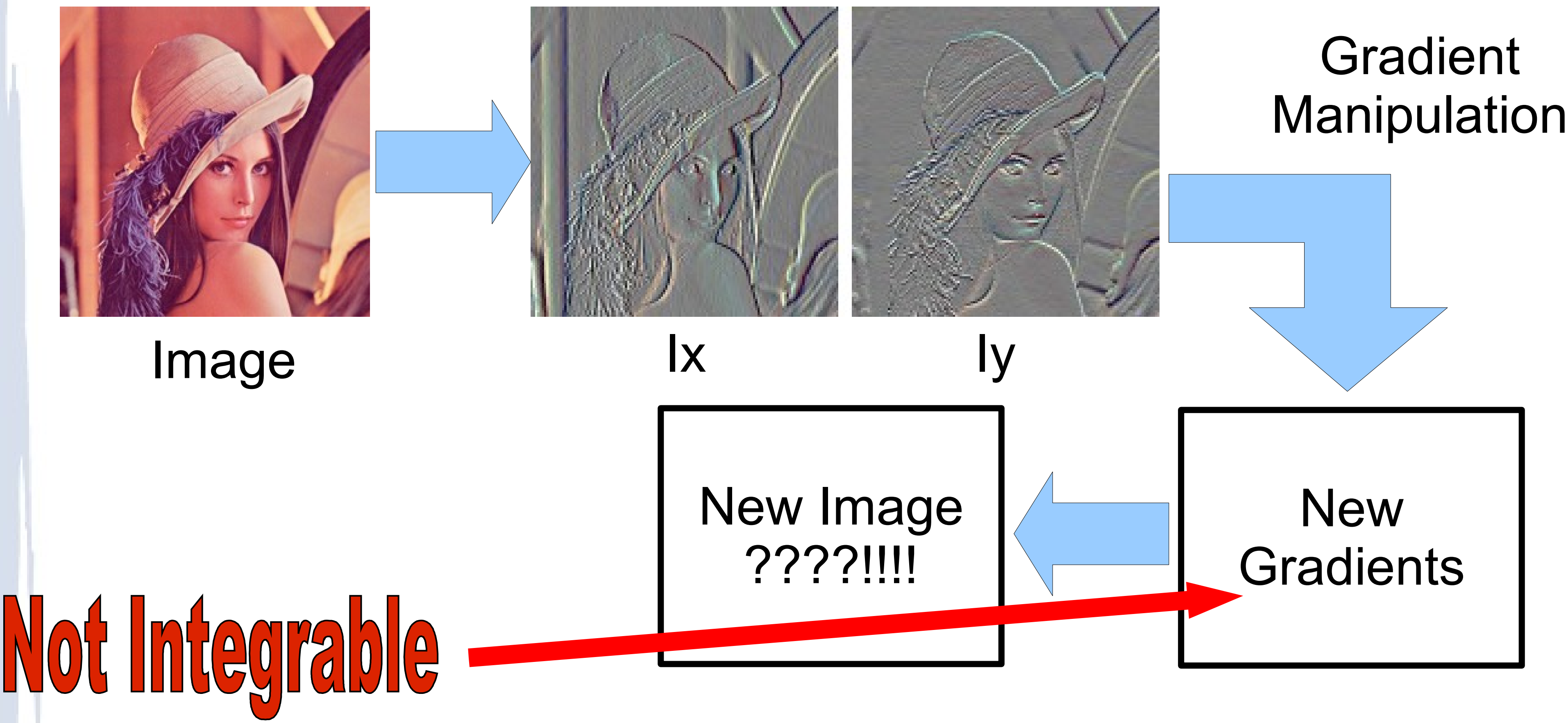


Scalar- vs. Gradient-field (3)

- Gradient-field (vector-field)
  - Every point is associated with a vector
    - Magnitude : rate of intensity change
    - Phase : direction of the maximum change

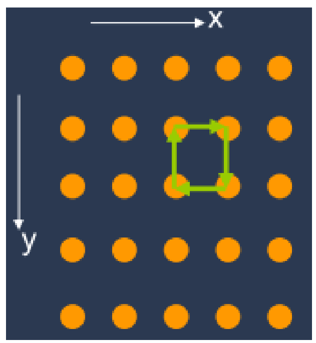
Poisson Equations

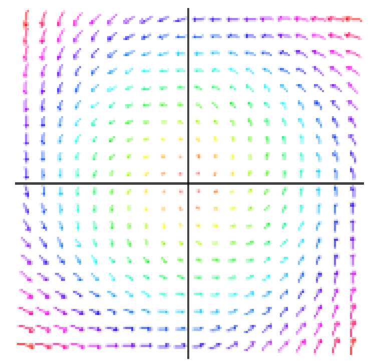
- Partial differential equations
- Why in the world we would need them ?!



Non-Integrability

- Condition of integrability
  - $I_x(x,y) + I_y(x+1,y) - I_x(x,y+1) - I_y(x,y) = 0$

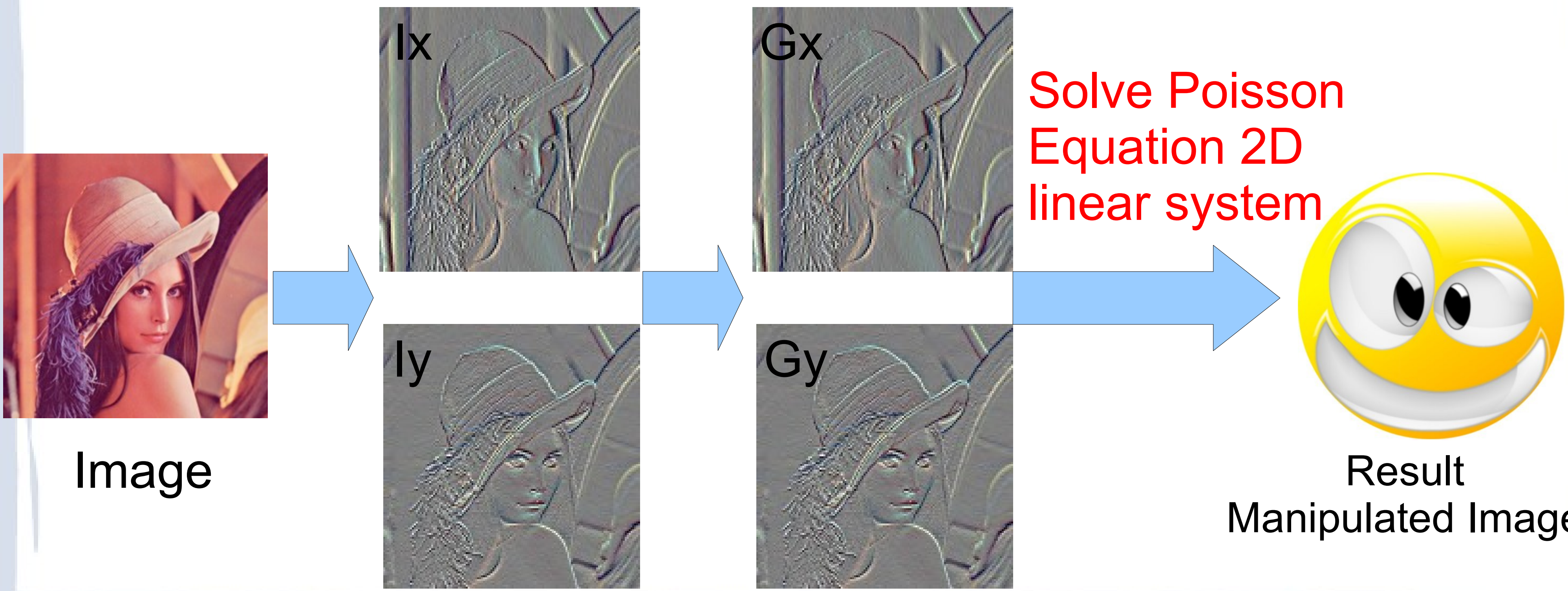




- This is called “Curl” property of the gradient
  - Shows rotation rate of a vector field

Poisson Equations (2)

- So, how to reconstruct the result Image ?!
  - Given : G(x,y) = Gx, Gy ; manipulated gradients
  - Compute : I(x,y)



Poisson Equations (3)

- We look for an Image I(x,y), whose gradient is close enough to G(x,y)

Minimize $\iint F(\nabla I, G)\, dxdy$

Where $F(\nabla I, G) = \|\nabla I - G\|^2$

close enough as quadratic distance

Poisson Equations (4)

mathematically we get

$$\nabla^2 I = d(Gx, Gy) = \frac{\partial Gx}{\partial x} + \frac{\partial Gy}{\partial y} = u(x, y)$$

For a numerical solution, we benefit from knowing that ∇^2 is the laplacian operator

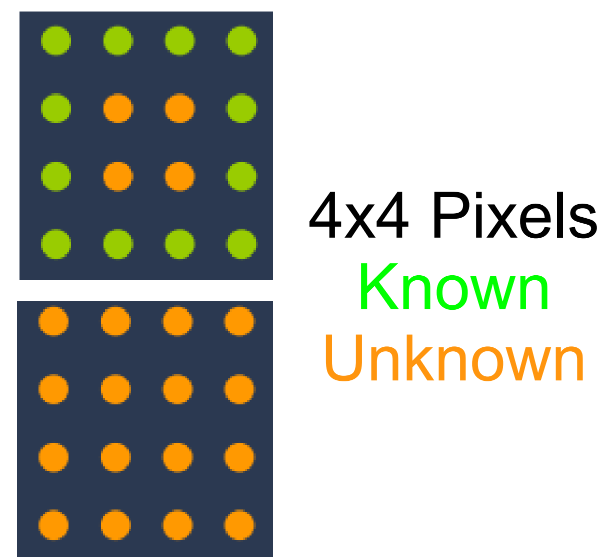
0	1	0
1	-4	1
0	1	0

Solving Poisson Equations

Substituting ∇^2 we came to a linear equation

$$Ax = b$$

And this is Solvable under 1 of 2 boundary conditions (Dirichlet & Neumann)



1. Boundaries are known (Dirichlet)

2. Boundaries are not known,

   but derivative normals instead (Neumann)

Solving Poisson Equations (2)

- Under both conditions we have a linear system solving problem
  - Direct Solver
    - Best approach for solving Poisson equation ***if*** rectangular boundary
    - Basic Idea : decompose A (SVD)
  - Basis Functions
  - Multigrid
  - Conjugate Gradients

Thank you :)

Any questions ??!!

References:

Agrawal A. and Raskar R. 2007. *Gradient Domain Manipulation Techniques in Vision and Graphics*

Hellwich O. 2009. *Digital Image Processing course*

www.wikipedia.org